

/* Error Correction Model */

[Linear Model with an AR Error Correction]

$$y_t = x_t' \beta + v_t$$

$$v_t = -\phi_1 v_{t-1} - \phi_2 v_{t-2} \cdot \cdot \cdot - \phi_p v_{t-p} + \epsilon_t$$

$$\epsilon_t \sim N(0, \sigma^2)$$

ϕ_i : AR parameters

[Linear Model with Heteroscedastic Error Terms]

$$y_t = x_t' \beta + \epsilon_t$$

$$\epsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \sigma^2 h_t$$

$$h_t = l(z_t' \eta)$$

cf) z_t : variables listed in the HETERO statement

η : parameter vector

$l(\cdot)$: link function

ex) link = exp : $h_t = \exp(z_t' \eta)$

link = square : $h_t = (1 + z_t' \eta)^2$

link = linear : $h_t = (1 + z_t' \eta)$

[Linear Model with ARCH-GARCH Error Terms]

$$y_t = x_t' \beta + v_t$$

$$v_t = -\phi_1 v_{t-1} - \phi_2 v_{t-2} \cdot \cdot \cdot - \phi_p v_{t-p} + \epsilon_t \text{ : Error Correction}$$

$$\epsilon_t = \sqrt{h_t} e_t$$

$$h_t = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \gamma_j h_{t-j} \text{ : GARCH(p,q)}$$

$$e_t \sim N(0, 1)$$

ex) the most basic ARCH: q=1, p=0

→ only 1 lagged squared residual is used to estimate the changing residual variance

ex) the basic GARCH(q=1,p=1)

→ 1 lagged squared residual and the most recent past estimate of the residual

variance are used to estimate the current residual variance